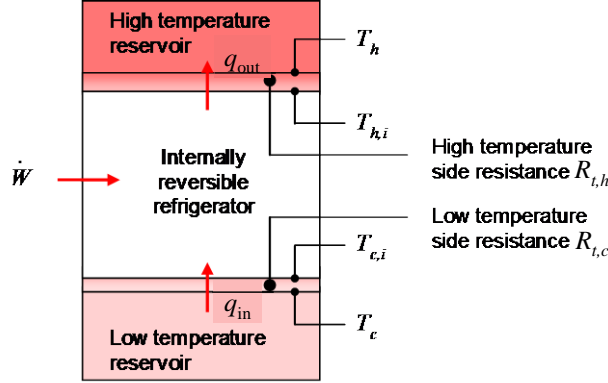


PROBLEM 1.32

KNOWN: Hot and cold reservoir temperatures of an internally reversible refrigerator. Thermal resistances between refrigerator and hot and cold reservoirs.

FIND: Expressions for modified Coefficient of Performance and power input of refrigerator.

SCHEMATIC:



ASSUMPTIONS: (1) Refrigerator is internally reversible, (2) Steady-state operation.

ANALYSIS: Heat is transferred from the low temperature reservoir (the refrigerated space) at T_c to the refrigerator unit, through the resistance $R_{t,c}$, with $T_c > T_{c,i}$. Heat is rejected from the refrigerator unit to the higher temperature reservoir (the surroundings), through the resistance $R_{t,h}$, with $T_{h,i} > T_h$. The heat input and output rates can be expressed in a manner analogous to Equations 1.18a and 1.18b.

$$q_{\text{in}} = (T_c - T_{c,i}) / R_{t,c} \quad (1)$$

$$q_{\text{out}} = (T_{h,i} - T_h) / R_{t,h} \quad (2)$$

Equations (1) and (2) can be solved for the internal temperatures, to yield

$$T_{h,i} = T_h + q_{\text{out}} R_{t,h} = T_h + q_{\text{in}} R_{t,h} \left(\frac{1 + \text{COP}_m}{\text{COP}_m} \right) \quad (3)$$

$$T_{c,i} = T_c - q_{\text{in}} R_{t,c} \quad (4)$$

In Equation (3), q_{out} has been expressed as

$$q_{\text{out}} = q_{\text{in}} \left(\frac{1 + \text{COP}_m}{\text{COP}_m} \right) \quad (5)$$

using the definition of COP_m given in the problem statement. The modified Coefficient of Performance can then be expressed as

Continued...

PROBLEM 1.32 (Cont.)

$$\text{COP}_m = \frac{T_{c,i}}{T_{h,i} - T_{c,i}} = \frac{T_c - q_{\text{in}} R_{t,c}}{T_h + q_{\text{in}} R_{t,h} \left(\frac{1 + \text{COP}_m}{\text{COP}_m} \right) - T_c + q_{\text{in}} R_{t,c}}$$

Manipulating this expression,

$$(T_h - T_c + q_{\text{in}} R_{t,c}) \text{COP}_m + q_{\text{in}} R_{t,h} (1 + \text{COP}_m) = T_c - q_{\text{in}} R_{t,c}$$

Solving for COP_m results in

$$\text{COP}_m = \frac{T_c - q_{\text{in}} R_{\text{tot}}}{T_h - T_c + q_{\text{in}} R_{\text{tot}}} \quad <$$

From the definition of COP_m , the power input can be determined:

$$\dot{W} = \frac{q_{\text{in}}}{\text{COP}_m} = q_{\text{in}} \frac{T_h - T_c + q_{\text{in}} R_{\text{tot}}}{T_c - q_{\text{in}} R_{\text{tot}}} \quad <$$

COMMENTS: As q_{in} or R_{tot} goes to zero, the Coefficient of Performance approaches the maximum Carnot value, $\text{COP}_m = \text{COP}_C = T_c / (T_h - T_c)$.